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Evolution of the two-point correlation function in the Zel'dovich approximation

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ABSTRACT

We study the evolution of the mass autocorrelation function by describing the growth of density fluctuations through the Zel'dovich approximation. The results are directly compared with the predictions of the scaling hypothesis for clustering evolution extracted from numerical simulations (Hamilton et al. 1991), as implemented by Jain, Mo & White (1995). We find very good agreement between the correlations on mildly non–linear scales and on completely linear scales. In between these regimes, we note that the density fields evolved through the Zel'dovich approximation show more non–linear features than predicted by the scaling ansatz which is, however, forced to match the linear evolution on scales larger than the simulation box. In any case, the scaling ansatz by Baugh & Gaztañaga (1996), calibrated against large box simulations agrees better with ZA predictions on large scales, keeping good accuracy also on intermediate scales.

We show that mode–coupling is able to move the first zero crossing of $\xi(r)$ as time goes on. A detailed fit of the time dependence of this shifting is given for a CDM model. The evolution of the cross correlation of the density fluctuation field evaluated at two different times is also studied. The possible implications of the results for the analysis of the observed correlation function of high redshift galaxies are discussed.

Key words: galaxies: clustering – cosmology: theory – large-scale structure of Universe

1 INTRODUCTION

In the last two decades, redshift surveys provided a wealth of informations about the spatial distribution of local galaxies, revealing the existence of large–scale structures. The most widely used statistical tool to quantify the degree of clustering has been the galaxy two–point correlation function, both in its angular $w_g(\theta)$ and spatial $\xi_g(r)$ versions (see, e.g., Peebles 1980). As previous studies were confined to the nearby universe, nowadays new observational resources permit to extend the correlation analysis to deeper samples. In fact, the Canada–France Redshift Survey has recently provided the new opportunity to investigate the clustering properties of galaxies out to redshifts $z \sim 1$ (Le Fèvre et al. 1996). Moreover, only lately it has been possible to analyse the angular distribution of faint

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galaxies by using the Hawaii Keck K–band survey (Cowie et al. 1996) and the Hubble Deep Field data (Villumsen, Freudling & da Costa 1997). Therefore, we are now confident of detecting a direct signature of redshift dependence in the observed correlation function. For this reason, the theoretical analysis of the evolution of the mass two–point correlation function, $\xi(r)$, is becoming a fundamental topic of modern cosmology. However, it is worth stressing that the interpretation of the observational data is not immediate: before obtaining the "real" change of the large–scale structure one has to consider the possible evolution of the galaxy population (as well as the related selection effects) and of the bias factor that formally relates ξ_g to ξ (see, e.g., Matarrese et al. 1997).

The observational results should then be compared to the predictions of the existing models for structure formation. One of the several issues involved in this comparison is represented by the lack of a standard description of clustering evolution: analytical treatments are generally unable to manage this fully non–linear problem while numerical simulations are limited in resolution. However, new light has been recently shed on this argument. Hamilton et al. (1991) suggested that the correlation function obtained through N-body simulations of an Einstein–de Sitter universe, in which the structure develops hierarchically, can be easily reproduced by applying a non–local and non–linear transformation to the linear $\xi(r)$. This ansatz has been refined and extended to more general cosmological scenarios by a number of authors (Peacock & Dodds 1994, Jain, Mo & White 1995, Peacock & Dodds 1996). Moreover it is possible to give theoretical arguments that account for the scaling hypothesis (Nityananda & Padmanabhan 1994).

The main purpose of this paper is to compare the predictions of the Zel'dovich approximation (Zel'dovich 1970) with the scaling ansatz formulated in the version of Jain, Mo & White (1995, hereafter JMW). Actually, it would be very interesting to obtain all the details of the semi–empirical scaling relationship in the framework of the gravitational instability scenario. However, in the absence of a model for the advanced phases of clustering evolution, we are forced to analyse only the onset of non–linear dynamics.

From the theoretical point of view, the evolution of the two–point correlation function is strictly related to the dynamical development of the density field $\varrho(\mathbf{x},t)$. When the dimensionless density contrast $\delta(\mathbf{x},t)=[\varrho(\mathbf{x},t)-\bar{\varrho}(t)]/\bar{\varrho}(t)$ is much smaller than unity, the growth of the fluctuations can be followed performing a perturbative approach (see, e.g., Peebles 1980, Fry 1984, Scoccimarro & Frieman 1996a). At the lowest order (linear theory), the different Fourier modes of $\delta(\mathbf{x},t)$ evolve independently provided that their power spectrum is less steep than k^4 at small k (Zel'dovich 1965, Peebles 1974). As the fluctuations grow, however, the interactions between different modes become more and more important. The effect of this mode–coupling on the two–point statistics has been studied by many authors using higher–order than linear terms in the perturbation expansion. Juskiewicz, Sonoda & Barrow (1984) computed the second–order contribution to $\xi(r)$ for an exponentially smoothed linear spectrum $P(k)\propto k^2$, finding that non–linear interactions among long wavelength modes act as a source for short λ perturbations. As a matter of fact, they found a substantial decrease of the characteristic scale of clustering with the evolution. However Suto & Sasaki (1991) and Makino, Sasaki & Suto (1992), analysing exponentially filtered scale–free spectra, found that second–order effects can either suppress or enhance the growth of perturbations on large scales, depending on the shape and the amplitude of the fluctuation spectrum. In particular Makino, Sasaki & Suto (1992), modelling a CDM spectrum with two different power laws, concluded that the effects of mode–coupling are generally very small and completely negligible on scales $r \gtrsim 20\,h^{-1}$Mpc (where h denotes the Hubble constant in units of $100\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$). The second–order correction to the "true" linear CDM spectrum has been calculated by Coles (1990) who computed also the respective correlation function. The results show that, for moderate evolution, the large–scale distortions are of no importance, while later (for $\sigma_8 \gtrsim 1$, where σ_8 represents the *rms* linear mass fluctuation in spheres of radius $8\,h^{-1}$Mpc) non–linear effects can increase the clustering strength on scales $r>35\,h^{-1}$Mpc; for example, the first zero crossing of $\xi(r)$ can be significantly shifted with respect to linear predictions. Similar results were obtained by Baugh & Efstathiou (1994) who also found good agreement with the output of numerical simulations. However, Jain & Bertschinger (1994) pointed out that the perturbative approach is able to reproduce the N–body outcomes only at early times ($\sigma_8 \lesssim 0.5-1$). Moreover, the recent analysis applied to scale–free spectra by Scoccimarro & Friemann (1996b) showed that the validity of perturbation theory is restricted to a small range of spectral indices.

In this paper, we want to study the non–linear evolution of the mass autocorrelation function by describing the

growth of density fluctuations through the Zel'dovich approximation (hereafter ZA). In effect, Eulerian second–order perturbation theory may break down once the mass variance becomes sufficiently large. On the other hand, we know that ZA, especially in its "truncated" form, is able to reproduce fairly well the outcomes of N–body simulations even in the mildly non–linear regime (Melott, Pellman & Shandarin 1994). The main advantage of ZA over other dynamical approximations (for a recent review see, e.g., Sahni & Coles 1995) is that it permits analytical investigations ensuring at the same time good accuracy, at least for quasi–linear scales. The pioneering analysis by Bond & Couchmann (1988) showed that ZA is able to predict the shifting of the first zero crossing of the correlation function. In Section 3 we will give a detailed quantitative description of this effect. Other features of the mass two–point correlation function in ZA have been discussed by Mann, Heavens & Peacock (1993, hereafter MHP). Moreover, the related evolution of the power spectrum has been studied by Taylor (1993), Schneider & Bartelmann (1995) and Taylor & Hamilton (1996). These authors showed that ZA is able to describe the generation of small–scale power through mode coupling, at least at early times. Besides Fisher & Nusser (1996) and Taylor & Hamilton (1996) succeeded in computing the power spectrum also in redshift space.

This paper is organized as follows. In Section 2 we briefly introduce the Zel'dovich approximation while in Section 3 we compute the cross correlation function between the mass density field evaluated at two different times. The usual two–point correlation function is obtained as a particular case of this more general quantity. The redshift evolution of $\xi(r)$ in a CDM model is the last subject of Section 3. In Section 4 we compare the predictions of ZA with the scaling ansatz of JMW. In Section 5 we use our results to evaluate the correlation function of a collection of objects sampled by an observer in a wide redshift interval of his past light cone. We then propose a simplified scheme to compute this quantity so as to improve another approximation presented in the literature. A brief summary is given in Section 6.

2 THE ZEL'DOVICH APPROXIMATION

Let us consider a set of collisionless, self–gravitating particles in an expanding universe with scale factor $a(t)$. We can describe the motion of each point–like particle writing its actual (Eulerian) comoving position, $\mathbf{x}$, at time t as the sum of its initial (Lagrangian) comoving position, $\mathbf{q}$, plus a displacement:

$$\mathbf{x}(\mathbf{q},t) = \mathbf{q} + \mathbf{S}(\mathbf{q},t). \tag{1}$$

The displacement vector field $\mathbf{S}(\mathbf{q},t)$ represents the effect of density perturbations on the trajectories. The Zel'dovich approximation is obtained by assuming the separability of the temporal and spatial parts of $\mathbf{S}(\mathbf{q},t)$ and by requiring equation (1) to give the correct evolution of $\delta(\mathbf{x},t)$ in the linear regime. Considering only the growing mode for a pressureless fluid, one gets (Zel'dovich 1970):

$$\mathbf{S}(\mathbf{q},t) = -b(t)\nabla\phi\Big|_{\mathbf{q}} \tag{2}$$

where $b(t)$ is the linear growth factor and $\phi(\mathbf{q})$ represents the initial peculiar velocity potential that at the linear stage is proportional to the gravitational potential $\Phi_0(\mathbf{q})$. The Zel'dovich approximation can be also extracted from a fully Lagrangian approach to the evolution of density fluctuations (Buchert 1989, Moutarde et al. 1991, Bouchet et al. 1992, Buchert 1993, Catelan 1995). In this case, ZA corresponds to the first order solution provided that the initial velocity field is irrotational and the initial peculiar velocity and acceleration fields are everywhere parallel.

Equations (1) and (2) define a mapping from Lagrangian to Eulerian space that develops caustics as time goes on (Shandarin & Zel'dovich 1989). However, the "Zel'dovich fluid" is a system with infinite memory: even after the intersection of two trajectories, the motion of the particles is determined by their initial conditions according to equation (2). The lack of self–gravity between intersecting streams causes the forming structure to be rapidly washed out. This is a severe problem especially in hierarchical models of structure formation, where caustics appear early on small scales causing ZA to become soon inaccurate. Nevertheless Coles, Melott & Shandarin (1993) showed that a modified version of ZA, the "truncated" ZA, obtained by smoothing the initial conditions, is able to reproduce with good accuracy the density distributions obtained from numerical simulations. Melott, Pellman & Shandarin (1994)

found that the optimal version of the truncation procedure is accomplished by using a Gaussian window to smooth the linearly extrapolated power spectrum of the density fluctuation field $b^2(t)P(k)$: †

$$P_T(k,t) = b^2(t)P(k)\exp\left[-k^2R_f^2(t)\right] \tag{3}$$

where the filtering radius $R_f(t)$ increases with time being related to the typical scale going non–linear. The success of this approximation can be justified by noticing that the non–linearly evolved gravitational potential resembles its smoothed linear counterpart (Pauls & Melott 1995). In the following we will adopt the filtering prescription given in equation (3).

3 THE TWO-POINT CORRELATION FUNCTION IN THE ZEL'DOVICH APPROXIMATION

Assuming that initially the mass is evenly distributed in Lagrangian space, implies that the Eulerian density field is related to the Lagrangian displacement field via the relation:

$$\varrho(\mathbf{x},t) = \bar{\varrho}(t)\int d^3q\,\delta_D\left[\mathbf{x}-\mathbf{q}-\mathbf{S}(\mathbf{q},t)\right], \tag{4}$$

where $\delta_D(\mathbf{x})$ denotes the three–dimensional Dirac delta function. For purposes that will be clarified in Section 5, we are interested in computing the cross correlation function between the density contrast field evaluated at two different times:

$$\langle\delta(\mathbf{x}_1,t_1)\delta(\mathbf{x}_2,t_2)\rangle = \langle\int d^3q_1d^3q_2\;\delta_D\left[\mathbf{x}_1-\mathbf{q}_1-\mathbf{S}(\mathbf{q}_1,t_1)\right]\delta_D\left[\mathbf{x}_2-\mathbf{q}_2-\mathbf{S}(\mathbf{q}_2,t_2)\right]\rangle - 1 \tag{5}$$

where $\langle\cdot\rangle$ represents the average over an ensemble of realizations. Before going any further, it is convenient to Fourier transform the Dirac delta functions in equation (5) obtaining:

$$1+\langle\delta(\mathbf{x}_1,t_1)\delta(\mathbf{x}_2,t_2)\rangle = \int d^3q_1d^3q_2\,\frac{d^3w_1}{(2\pi)^3}\,\frac{d^3w_2}{(2\pi)^3}\exp\left[i\sum_{j=1}^{2}\mathbf{w}_j\cdot(\mathbf{x}_j-\mathbf{q}_j)\right]\langle\exp\left[-i\sum_{\ell=1}^{2}\mathbf{w}_\ell\cdot\mathbf{S}(\mathbf{q}_\ell,t_\ell)\right]\rangle. \tag{6}$$

We then use equation (2) to introduce ZA into equation (6). In such a way, by assuming, as usual, that $\phi(\mathbf{q})$ is a statistically homogeneous and isotropic Gaussian field, uniquely specified by its power spectrum $P_\phi(k)\propto P(k)/k^4$, the ensemble average contained in equation (6) can be written as a functional integral:

$$\langle\exp\left[-i\sum_{\ell=1}^{2}\mathbf{w}_\ell\cdot\mathbf{S}(\mathbf{q}_\ell,t_\ell)\right]\rangle = (\det K)^{1/2}\int D[\phi]\exp\left[-\frac{1}{2}\int\phi(\mathbf{q})K(\mathbf{q},\mathbf{q}')\phi(\mathbf{q}')d^3qd^3q' + i\sum_{\ell=1}^{2}b(t_\ell)\mathbf{w}_\ell\cdot\nabla\phi\Big|_{\mathbf{q}_\ell}\right] \tag{7}$$

where the kernel $K(\mathbf{q},\mathbf{q}')$ represents the functional inverse of the two–point correlation function of the field $\phi(\mathbf{q})$. By defining a six–dimensional vector $\mathbf{c^t}=(\mathbf{w}_1,\mathbf{w}_2)$ and choosing the z-axis of our reference frame in the direction of the vector $\mathbf{q}=\mathbf{q}_1-\mathbf{q}_2$, we can reduce equation (7) to the form:

$$\langle\exp\left[-i\sum_{\ell=1}^{2}\mathbf{w}_\ell\cdot\mathbf{S}(\mathbf{q}_\ell,t_\ell)\right]\rangle = \exp\left[-\frac{1}{2}\mathbf{c^t}\mathbf{M}\mathbf{c}\right] \tag{8}$$

where the matrix $\mathbf{M}$ has the structure

$$\mathbf{M} = \gamma\begin{pmatrix} b_1^2 & 0 & 0 & b_1b_2\psi_\perp & 0 & 0 \\ 0 & b_1^2 & 0 & 0 & b_1b_2\psi_\perp & 0 \\ 0 & 0 & b_1^2 & 0 & 0 & b_1b_2\psi_\parallel \\ b_1b_2\psi_\perp & 0 & 0 & b_2^2 & 0 & 0 \\ 0 & b_1b_2\psi_\perp & 0 & 0 & b_2^2 & 0 \\ 0 & 0 & b_1b_2\psi_\parallel & 0 & 0 & b_2^2 \end{pmatrix} \tag{9}$$

with $b_i = b(t_i)$ and

† We set $b=1$ at the present epoch.

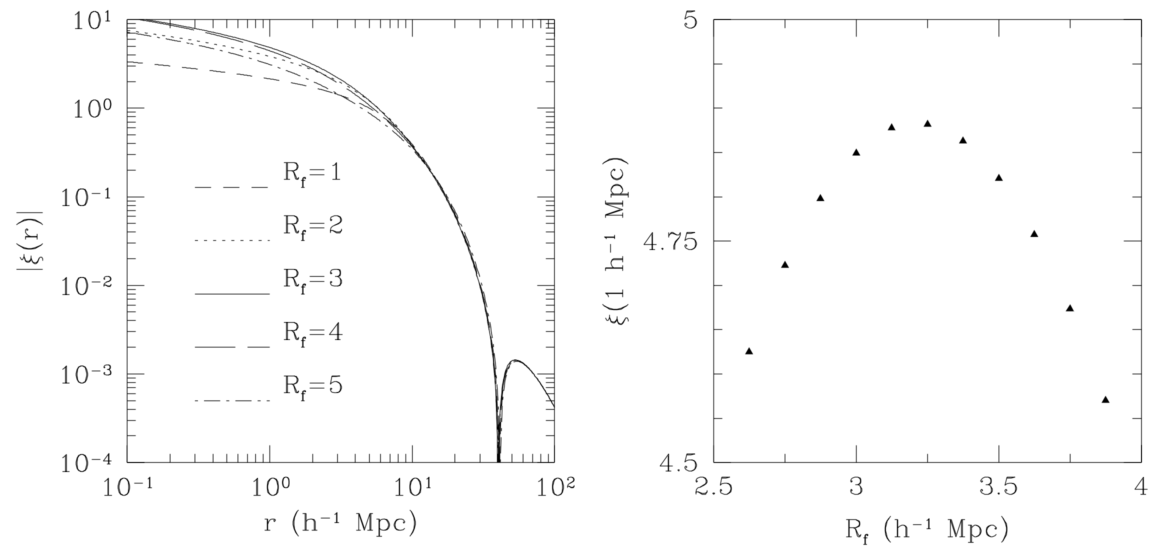

Figure 1. Left panel: the mass autocorrelation function, obtained using ZA, for a *COBE* normalized CDM linear spectrum is plotted for different values of the truncation radius R_f (in h^{-1}Mpc). Right panel: dependence of the correlation function evaluated at $r = 1\,h^{-1}$Mpc on R_f.

$$\gamma = \frac{1}{6\pi^2}\int_0^\infty P(k)dk\ , \quad \gamma\psi_\parallel(q) = \frac{1}{2\pi^2}\int_0^\infty P(k)\left[j_0(kq) - \frac{2}{kq}j_1(kq)\right]dk\ , \quad \gamma\psi_\perp(q) = \frac{1}{2\pi^2}\int_0^\infty P(k)\frac{1}{kq}j_1(kq)dk\ , \tag{10}$$

having denoted by $j_\ell(x)$ the spherical Bessel function of order ℓ. By substituting this result into equation (6) we can easily solve the Gaussian integration over the $\mathbf{w}_i$. In order to perform the remaining integrations, it is convenient to introduce the new variables $\mathbf{q}$ and $\mathbf{Q} = \mathbf{q}_1 + \mathbf{q}_2$. In this way, after some algebra, we finally obtain:

$$1 + \xi(r, t_1, t_2) \equiv 1 + \langle\delta(\mathbf{x}_1, t_1)\delta(\mathbf{x}_2, t_2)\rangle = \frac{1}{(2\pi)^{1/2}r}\int_0^\infty \frac{q^2 dq}{(b_1 b_2)^{1/2}\gamma(\psi_\perp - \psi_\parallel)^{1/2}(b_1^2 + b_2^2 - 2b_1b_2\psi_\perp)^{1/2}}\times$$
$$\times\left\{D(u_+)\exp\left[-\frac{(q-r)^2}{2\gamma(b_1^2 + b_2^2 - 2b_1b_2\psi_\parallel)}\right] - D(u_-)\exp\left[-\frac{(q+r)^2}{2\gamma(b_1^2 + b_2^2 - 2b_1b_2\psi_\parallel)}\right]\right\} \tag{11}$$

where $r = |\mathbf{x}_1 - \mathbf{x}_2|$,

$$u_\pm = \left[\frac{b_1b_2(\psi_\perp - \psi_\parallel)}{\gamma(b_1^2 + b_2^2 - 2b_1b_2\psi_\perp)(b_1^2 + b_2^2 - 2b_1b_2\psi_\parallel)}\right]^{1/2}\left[\frac{b_1^2 + b_2^2 - 2b_1b_2\psi_\perp}{2b_1b_2(\psi_\perp - \psi_\parallel)}q \pm r\right] \tag{12}$$

and $D(x)$ represents the Dawson's integral ‡ (see, e.g., Abramowitz & Stegun, 1968). It is straightforward to show that for $t_1 = t_2$ the previous formula reduces to the usual expression for the mass two–point correlation function in ZA (Bond & Couchman 1988, Mann, Heavens & Peacock 1993, Schneider & Bartelmann 1995).

We numerically evaluated the two–point correlation function $\xi(r, t) \equiv \xi(r, t, t)$ employing a *COBE* normalized standard CDM linear power spectrum (with density parameter $\Omega = 1$ and $h = 0.5$). We used the transfer function of Bardeen et al. (1986) while the normalization to the four–year *COBE* DMR data is given in Bunn & White (1997) and corresponds to $\sigma_8 = 1.22$. As already noted by MHP, the small scale behaviour of the resulting correlation function depends on the value assigned to the truncation radius, R_f, defined in equation (3) (see Fig. 1). If R_f is very small, then shell crossing will not be suppressed and $\xi(r)$ will show an unusually flat behaviour. On the contrary, if R_f is too large, the smoothing procedure will remove an important contribution to the power spectrum, causing again too

‡ It is worth stressing that when $\psi_\perp < \psi_\parallel$, in order to avoid a complex argument for the Dawson's integral, it is convenient to express the integrand in equation (11) in terms of exponentials and error functions (see also the discussion in Schneider & Bartelmann, 1995). However, since for the CDM spectrum (the only one considered in our analysis) $\psi_\parallel$ is never larger than $\psi_\perp$, we preferred to write the solution using $D(x)$.

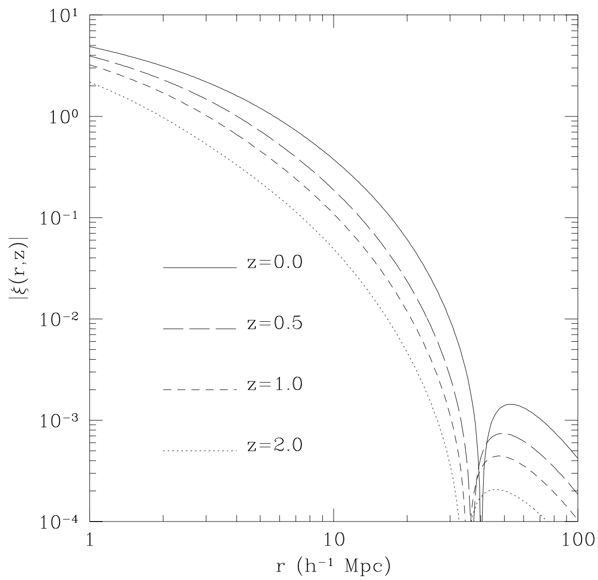

Figure 2. Redshift evolution of the mass two–point correlation function obtained using ZA to evolve a linear CDM spectrum.

low a correlation. Therefore we need a criterion to select R_f. Since our main purpose is to compare the clustering amplitudes predicted by ZA with those extracted from the scaling ansatz of JMW, we can choose R_f so as to optimize the agreement between the respective correlation functions. Anyway, we find that this method conforms quite well to a simpler one already used by MHP: the best R_f is the one that maximizes $\xi(r, R_f)$ on small scales. Strictly speaking, the optimal smoothing radius depends on the scale selected for maximizing the correlation: the smaller is r the larger comes out R_f (we find that the difference between the smoothing lengths obtained by maximizing ξ at $r = 0.1\, h^{-1}$Mpc and at $r = 1\, h^{-1}$Mpc roughly amounts to $0.2\, h^{-1}$Mpc and remains nearly constant by varying σ_8). However, the effect of this discrepancy on the correlation evaluated on larger scales is indeed minimal. Following Schneider & Bartelmann (1995), we select $r = 1\, h^{-1}$Mpc as the scale at which we require $\xi(R_f)$ to be maximal. As previously stated, the optimum filtering length increases as the field evolves; the dependence of the best R_f on σ_8 is almost linear and for $\sigma_8 > 0.3$ (that in our model corresponds to $z \sim 3$) it can be approximated by:

$$R_f(\sigma_8) = (3.16\, \sigma_8 - 0.65)\, h^{-1}\mathrm{Mpc}\ . \tag{13}$$

The redshift evolution of the correlation function is shown in Fig. 2. As expected, on scales that are not affected by shell crossing ($r > R_f$), $\xi(r, z)$ steepens with decreasing z. Moreover, we note that the first zero crossing radius of $\xi(r, z)$ increases as time goes on (see also Bond & Couchmann 1988). A similar pattern has been noticed by Coles (1990) and by Baugh & Efstathiou (1994) in the context of second–order Eulerian perturbation theory. The displacement of the first zero crossing of ξ as a function of time is plotted in Fig. 3. Measuring the degree of dynamical evolution of the density field through σ_8, this shifting can be described with good approximation by the function:

$$r_{0\mathrm{C}}(\sigma_8) - r_{0\mathrm{C}}^{\mathrm{lin}} \simeq 5.3\, \sigma_8^{(1.5+0.1/\sigma_8)}\, h^{-1}\mathrm{Mpc} \tag{14}$$

where we denoted by $r_{0\mathrm{C}}$ the scale at which the correlation function crosses for the first time the zero–level and by $r_{0\mathrm{C}}^{\mathrm{lin}}$ its linear counterpart. It would be interesting to compare this result with the predictions of second–order Eulerian (and Lagrangian) perturbation theory and of other dynamical approximations.

4 COMPARISON WITH THE SCALING HYPOTHESIS

The analysis of a large set of numerical simulations suggests that, in hierarchical models, the non–linear two–point correlation function, $\xi(r, z)$, can be related to the linear one, $\xi_{\mathrm{L}}(r, z)$, through a simple scaling relation (Hamilton

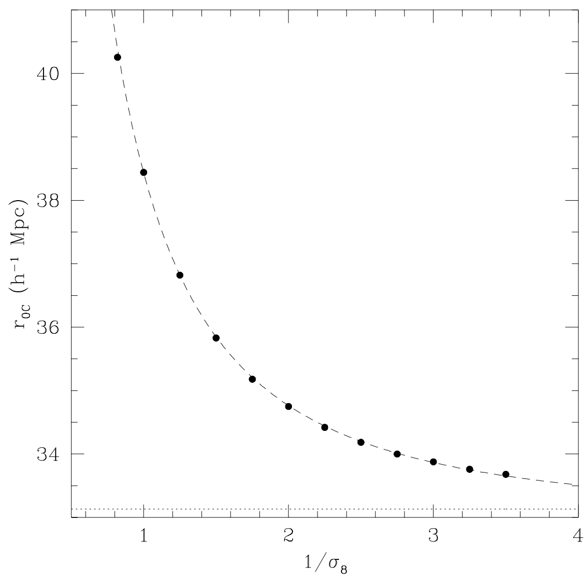

Figure 3. The first zero crossing radius of the correlation function is plotted against $1/\sigma_8$. The circles represent the results obtained using ZA, the dashed line is the fitting function given in the text while the dotted line shows the prediction of Eulerian linear theory.

et al. 1991, Peacock & Dodds 1994, Jain, Mo & White 1995, Peacock & Dodds 1996). The main idea is that the action of gravity can be represented as a continuous change of scale or, better, that the 'flow of information' about clustering propagates along the curves of equation:

$$r_0 = [1+\bar{\xi}(r,z)]^{1/3} r \ , \tag{15}$$

where $\bar{\xi}(r,z)$ represents the average correlation function within a sphere of radius r

$$\bar{\xi}(r,z) = \frac{3}{r^3}\int_0^r y^2 \xi(y,z)dy \tag{16}$$

and r_0 is a sort of Lagrangian coordinate determining a 'conserved pair surface' (Hamilton et al. 1991, Nityananda & Padmanabhan 1994). In fact, by definition, the average number of neighbours of a particle contained within a spherical volume of radius r_0 at the linear stage (when $\bar{\xi} \ll 1$) equals the average number of neighbours inside a sphere of radius r in the evolved field.

Here, we want to compare the results obtained in the previous section, using ZA, with the predictions of the scaling ansatz (hereafter SA) formulated in the version of JMW:

$$\bar{\xi}(r,z) = B(n_{\rm eff}) F\left[\frac{\bar{\xi}_{\rm L}(r_0,z)}{B(n_{\rm eff})}\right] \tag{17}$$

with

$$F(x) = \frac{x + 0.45x^2 - 0.02x^5 + 0.05x^6}{1 + 0.02x^3 + 0.003x^{9/2}} \ , \qquad B(n_{\rm eff}) = \left(\frac{3+n_{\rm eff}}{3}\right)^{0.8} \ , \qquad n_{\rm eff}(z) = \left.\frac{d\ln P(k)}{d\ln k}\right|_{k_{\rm NL}(z)} \tag{18}$$

where $k_{\rm NL}^{-1}$ denotes the radius of the top–hat window function in which the *rms* linear mass fluctuation is unity. However, it would be useless to perform the comparison between the spherically averaged correlation functions since, on small scales, $\xi(r)$ obtained using ZA is seriously affected by shell crossing and the computation of $\bar{\xi}$ requires an integration starting from $r = 0$. For this reason we prefer to use directly $\xi(r)$. The two–point correlation function deriving from the ansatz of JMW can be obtained performing a simple differentiation:

$$\xi(r,z) = \frac{\left[1 + B(n_{\rm eff})F(X)\right]F'(X)\Delta\xi_{\rm L}(r_0,z)}{1 + B(n_{\rm eff})F(X) - F'(X)\Delta\xi_{\rm L}(r_0,z)} + B(n_{\rm eff})F(X) \ , \qquad X = \frac{\bar{\xi}_{\rm L}(r_0,z)}{B(n_{\rm eff})} \ , \tag{19}$$

with $F'(x) = dF/dx$ and

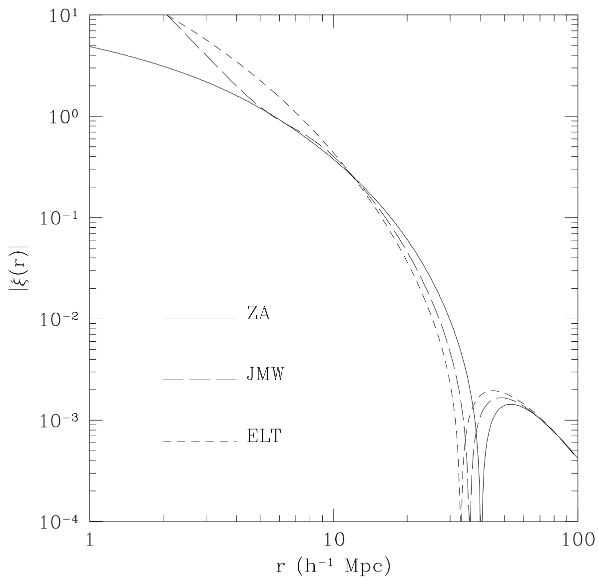

Figure 4. Comparison between the mass autocorrelation functions computed for a CDM model by using: the Zel'dovich approximation (ZA), the scaling ansatz of Jain, Mo & White (JMW) and Eulerian linear theory (ELT). The linear power spectrum extrapolated to the present epoch ($z = 0$) is normalized to match the *COBE* DMR data ($\sigma_8 = 1.22$).

$$\Delta\xi_{\rm L}(r_0, z) \equiv \xi_{\rm L}(r_0, z) - \bar{\xi}_{\rm L}(r_0, z) = \frac{b^2(z)}{2\pi^2}\int_0^\infty k^2 P(k)\big[j_0(kr_0) - \frac{3}{kr_0}j_1(kr_0)\big]\,dk\ . \tag{20}$$

We evaluated the correlation function given in equation (19) using a *COBE* normalized, linear CDM spectrum. In Fig. 4 we plot the result obtained at $z = 0$ with the corresponding one achieved by using ZA. For comparison we also show the prediction of Eulerian linear theory. The agreement between ZA and SA is remarkable on mildly non–linear scales ($4\,h^{-1}$Mpc$\lesssim r \lesssim 20\,h^{-1}$Mpc) and on completely linear scales ($r > 50\,h^{-1}$Mpc). For example, at $r = 5\,h^{-1}$Mpc, linear theory overestimates the correlation of JMW by 82%, ZA underestimates it by 2% while the accuracy of the JMW fit is about $15-20$%. However, we find that in the interval $20\,h^{-1}$Mpc$\lesssim r \lesssim 50\,h^{-1}$Mpc ZA predicts more non–linear evolution than SA (for example the $r_{\rm 0C}$ obtained by using ZA is larger than the one determined through SA). In order to consider a less evolved field, in Fig. 5 we repeat the comparison using the correlation functions evaluated at $z = 1$. Now, the main item to note is that the JMW result matches the linear solution on scales ($r \sim 10\,h^{-1}$Mpc) that, according to ZA, are already involved in non–linear phenomena.

In any case, we do not know the accuracy of the scaling hypothesis on large scales. In fact, the function $F(x)$ is obtained by requiring the resulting $\xi(r)$ to reproduce the linear behaviour where $\bar{\xi}_{\rm L} \to 0$ and, simultaneously, to approximate properly the correlation function extracted from N–body simulations. However, in order to achieve a detailed description of non–linear scales, JMW used a relatively small box to perform their simulations. Therefore, imposing the match to linear theory on large scales, without having any constraint from numerical data on quasi–linear scales, could seriously alter the accuracy of $F(x)$. This probably implies that the JMW fitting function could be improved on large scales. Our conclusion is shared by Baugh & Gaztañaga (1996, hereafter BG), who tested the scaling ansatz for the evolution of the power spectrum against the results of 5 N–body simulations performed within a $378\,h^{-1}$Mpc box. Indeed, they found that the JMW formula gives a relatively poor description of the large–scale behaviour even though the agreement between the spectra remains always within the quoted 20 % accuracy.

By using the output of their simulations, BG proposed a new scaling formula calibrated on large scales. As initially suggested by Peacock & Dodds (1994), the analytic expression of this SA concerns the dimensionless power spectrum, $\Delta^2(k, z) = k^3 P(k, z)/2\pi^2$ (i.e. the contribution to the the variance of the density contrast per bin of $\ln k$), while, following JMW, it takes account of a spectral dependence of the transformation:

$$\Delta^2(k, z) = \beta(n_{\rm eff}) f\left[\frac{\Delta^2_{\rm L}(k_{\rm L}, z)}{\beta(n_{\rm eff})}\right]\ , \qquad k_{\rm L} = \left[1 + \Delta^2(k, z)\right]^{-1/3} k \tag{21}$$

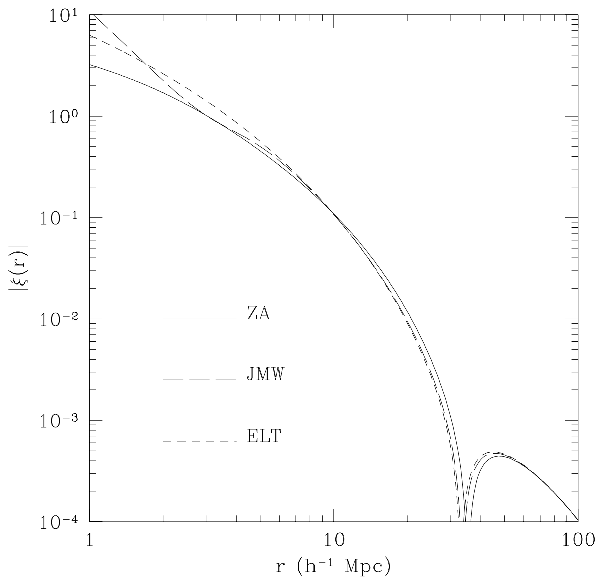

Figure 5. As in Fig. 4, but at $z = 1$ ($\sigma_8 = 0.61$).

where

$$f(x) = x\left(\frac{1 + 0.598x - 2.39x^2 + 8.36x^3 - 9.01x^{3.5} + 2.895x^4}{1 - 0.424x + [2.895/(11.68)^2]x^3}\right)^{1/2}, \qquad \beta(n_{\rm eff}) = 1.16\left(\frac{3 + n_{\rm eff}}{3}\right)^{1/2} \tag{22}$$

and the subscript L marks linear quantities. The function $f(x)$ has been obtained by matching the power spectrum in the simulations at $\sigma_8 = 1$, with an accuracy of 5%, over the range $0.02\,h\,\mathrm{Mpc}^{-1} < k < 1.0\,h\,\mathrm{Mpc}^{-1}$ and by forcing the fit to have the asymptotic form $f(x) \to 11.68\,x^{3/2}$ when $x \to \infty$ (Hamilton *et al.* 1991). The two–point correlation function is related to $\Delta^2(k, z)$ through the Fourier relation:

$$\xi(r, z) = \int_0^\infty \Delta^2(k, z) j_0(kr) \frac{dk}{k}. \tag{23}$$

In Fig. 6 we compare the correlations obtained by using ZA and the JMW formula with the results of the scaling ansatz by BG: we are considering a standard CDM linear spectrum at the epoch in which $\sigma_8 = 1.22$. We immediately note that using larger simulation boxes to calibrate the SA allows a better determination of the correlation function for $r \gtrsim 20\,h^{-1}$Mpc. In fact, we find that the correlations obtained with ZA and with the BG formula agree by better than 20% for $r > 4.6\,h^{-1}$Mpc (with the exception of a very small r-interval centred in the first zero crossing of ξ) while the discrepancy between ZA and the JMW ansatz is less than 20% over the ranges $4.1\,h^{-1}\mathrm{Mpc} < r < 18.3\,h^{-1}$Mpc and $r > 49.6\,h^{-1}$Mpc. Similar patterns are obtained considering different values of σ_8. This shows that the BG fit, that has been calibrated against large box CDM simulations, gives also a very good description of the mass clustering predicted by ZA on intermediate scales. In any case, as expected, the JMW formula is sensibly more accurate for $5\,h^{-1}\mathrm{Mpc} \lesssim r \lesssim 15\,h^{-1}$Mpc where the BG predictions grow worse as σ_8 assumes values significantly larger than 1.

On the other hand, it would be interesting to check the reliability of ZA and second–order Eulerian perturbation theory by directly comparing their predictions on these scales. Bond & Couchmann (1988), studying the weakly non–linear evolution of the CDM power spectrum, found remarkable agreement between the two approximations. Moreover, Baugh & Efstathiou (1994) showed that second–order Eulerian perturbation theory can reproduce, at least qualitatively, the evolution of the power spectrum predicted by numerical simulations. However, Jain & Bertschinger (1994) found that the agreement between perturbation theory and N–body outcomes gets worse as the density field evolves. Besides, their results are inconsistent with the low–k behaviour of the second–order Eulerian correction to the CDM power spectrum computed by Bond & Couchmann (1988), raising again the issue about the compatibility between ZA and perturbation theory. In a recent work concerning the evolution of scale invariant spectra, Scoccimarro & Friemann (1996b) showed that, if the spectral index n satisfies $-3 < n < -1$, Eulerian perturbation theory is

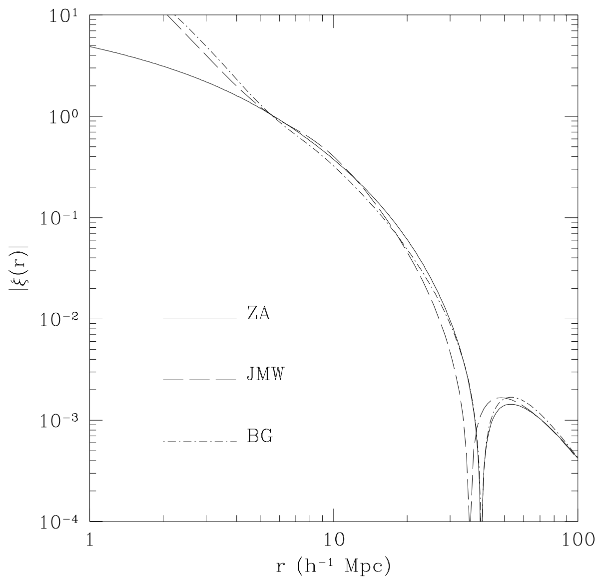

Figure 6. Mass two–point correlation functions at the epoch in which $\sigma_8 = 1$ obtained from a linear CDM spectrum evolved through the Zel'dovich approximation (ZA) and through the scaling ansätze by Jain, Mo & White (JMW) and by Baugh & Gaztañaga (BG).

able to reproduce fairly well the power spectrum obtained though the scaling ansatz, while the one–loop perturbative version of ZA gives worse results. Anyway, Bharadwaj (1996a,b) pointed out that the effects of multistreaming on the correlation function cannot be studied perturbatively. This fact implies that our result, obtained considering the full Zel'dovich approximation, should be more reliable than any other achieved by adopting a perturbative version of ZA. In any case, it would be interesting to clarify to which extent ZA and Eulerian perturbation theory agree on large scales.

5 THE CORRELATION OF HIGH REDSHIFT OBJECTS

In this section, we study the evolution of the cross correlation function of the mass density contrast evaluated at two different times as defined in equation (11). This quantity could play an important role in comparing the clustering properties extracted from deep redshift surveys to the predictions of theoretical models for structure formation. In practice, one always collects data on correlations in a finite redshift strip of his past light cone while the quantity $\xi(r, t)$, normally used in theoretical works, refers to objects selected on an hypersurface of constant cosmic time. Therefore, as far as one is considering a deep sample of cosmic objects, it is not correct to relate the observed clustering properties to $\xi(r, t)$. This issue is addressed in detail by Matarrese et al. (1997, hereafter MCLM) who build a theoretical quantity that allows a direct comparison of model predictions to the observed correlations. Their approach can be divided into three steps: first of all they compute the redshift evolution of mass correlations, then they relate the clustering properties of cosmic objects to the matter distribution by means of a linear bias relationship and finally they convolve the result with the observed redshift distribution of the class of objects under analysis. By assuming that the effects of redshift distortions and of the magnification bias due to weak gravitational lensing are negligible and by considering isotropic selection functions, MCLM showed that the theoretical estimate for the observed two–point correlation function can be formally expressed as an integral over z_1 and z_2 of the function $\xi(r, z_1, z_2)$ weighted by geometrical factors and effective bias parameters (all dependent on z_1 and z_2). Different classes of objects are selected by changing the amplitude and the redshift dependence of the effective bias. However, in the absence of a model for the evolution of the cross–correlation, only assuming that the above mentioned integral is dominated by the contribution of objects whose redshifts are nearly the same, can one estimate the observed correlation function deriving from a particular scenario of structure formation. In this way, one is allowed to replace $\xi(r, z_1, z_2)$ with $\xi(r, \bar{z})$,

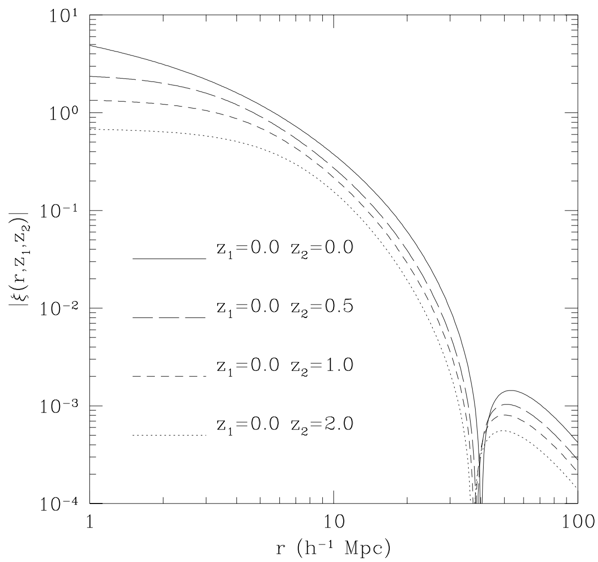

Figure 7. Cross correlation between the density contrast field evaluated at two different redshifts vs. comoving separation.

where $\bar{z}$ is a suitably defined average between z_1 and z_2 that, for simplicity, MCLM identify with $\bar{z} = (z_1 + z_2)/2$. This is a crucial approximation, as it allows MCLM to use the JMW ansatz to compute the non linear mass correlation function (there is no known scaling ansatz for $\xi(r, z_1, z_2)$). However, as shown in the previous paragraphs, ZA allows the computation of $\xi(r, z_1, z_2)$ so that we are able to compute the theoretical estimate for the observed correlation function by using both the complete and the approximated formulae given by MCLM (respectively their equations 15 and 18). Therefore we can check here, within the validity of ZA, the reliability of the approximation introduced by MCLM. Large discrepancies between the exact and the approximated correlations would obviously invalidate their whole analysis and consequently also their complete formula for $\xi_{\rm obs}$ would be unutilizable. On the other hand, if the approximated correlation function turns out to reproduce accurately the complete one, MCLM formulae could represent an important tool to disprove cosmological models in the light of present and future observations.

In order to compute $\xi(r, z_1, z_2)$ using equation (11), we truncated the linearly extrapolated power spectrum $b(z_1)b(z_2)P(k)$ according to the prescription:

$$P_T(k, z_1, z_2) = b(z_1)b(z_2)P(k)\exp\left[-k^2 R_f(z_1)R_f(z_2)\right] \tag{24}$$

where $R_f(z)$ represents the optimum filtering length for the density field at redshift z, determined by following the method described in Section 3. On small scales, the correlation functions that we obtain opting for this truncation procedure appear much more flattened than those computed at a single time. The evolution of $\xi(r, z_1, z_2)$ as z_2 changes is shown, for a CDM model, in Fig. 7. It is evident that even though the correlation decreases as z_2 grows, its decay is very slow. Actually, the ratios between the correlations computed at the same r, for different pairs of redshifts, are very similar to the predictions of linear theory. We find that the redshift evolution of the cross correlation function can be approximately described by the relation:

$$\xi(s, z_1, z_2) \simeq [\xi(s, z_1)\xi(s, z_2)]^{1/2}\,[1 - 2\Theta(s-1)] \tag{25}$$

where the quantity $s = r/r_{0\rm C}(z)$ is introduced in order to take into account the shifting of the first zero crossing of $\xi(r, z)$ and $\Theta(x)$ is the Heaviside step function. Moreover, the first zero crossing radius of $\xi(r, z_1, z_2)$ is nearly given by the geometric average of $r_{0\rm C}(z_1)$ and $r_{0\rm C}(z_2)$. For $s > 0.1$ equation (25), which is meaningful up to the scale at which the first of the two $\xi(s, z)$ reaches its second zero crossing, reproduces $\xi(s, z_1, z_2)$ with an accuracy of $\sim 5\%$. Anyway, for $s \gtrsim 2$, the usual relation $\xi(r, z_1, z_2) \simeq [\xi(r, z_1)\xi(r, z_2)]^{1/2}\mathrm{sign}[\xi(r, z_1)]$ deriving from linear theory is preferable.

We can now check the accuracy of the approximation introduced by MCLM that consists in computing the

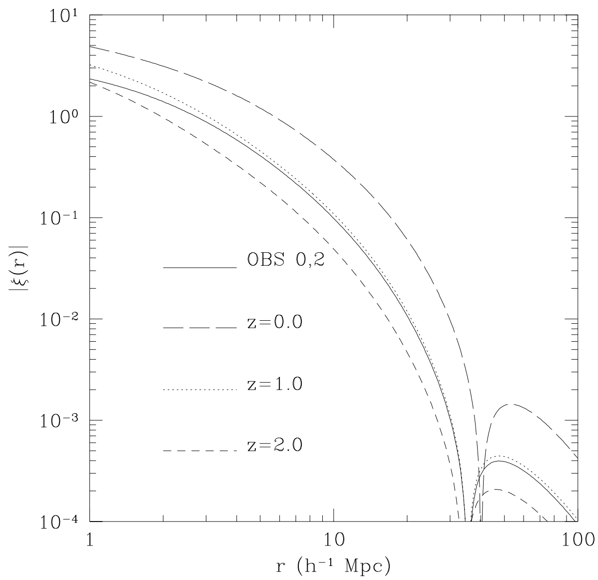

Figure 8. The observed two–point correlation function computed using equation (26) for a CDM model with $[z_{\rm min}, z_{\rm max}] = [0, 2]$. For comparison, the corresponding $\xi(r, z)$ evaluated for $z = 0, 1, 2$ are plotted.

theoretical estimate for the observed correlation function by replacing $\xi(r, z_1, z_2)$ with $\xi(r, \bar{z})$, where $\bar{z} = (z_1 + z_2)/2$, in the appropriate formula. For simplicity (and in order to isolate the phenomenon of clustering evolution) we will assume no bias, no selection effects and a constant comoving number density in an Einstein–de Sitter universe. In this case, equation 15 of MCLM reduces to:

$$\xi_{\rm obs}(r, z_{\rm min}, z_{\rm max}) = \frac{\displaystyle\int_{z_{\rm min}}^{z_{\rm max}} \frac{2 + z_1 - 2(1 + z_1)^{1/2}}{(1 + z_1)^{5/2}} \, \frac{2 + z_2 - 2(1 + z_2)^{1/2}}{(1 + z_2)^{5/2}} \, \xi(r, z_1, z_2) \, dz_1 dz_2}{\left[\displaystyle\int_{z_{\rm min}}^{z_{\rm max}} \frac{2 + z - 2(1 + z)^{1/2}}{(1 + z)^{5/2}} \, dz\right]^2} \tag{26}$$

where we denoted by $\xi_{\rm obs}(r, z_{\rm min}, z_{\rm max})$ the (ensemble averaged) theoretical estimate for the two–point correlation function measured by an observer that acquires data from the region of his past light cone corresponding to the redshift interval $[z_{\rm min}, z_{\rm max}]$.

Considering only the linear evolution of density fluctuations, $\xi(r, z_1, z_2) = \xi(r, 0, 0)/[(1+z_1)(1+z_2)]$, the integrals contained in equation (26) can be analytically performed. In this case, the quantity $\xi_{\rm obs}(r, z_1, z_2)/\xi(r, 0, 0)$ does not depend on r; for example we obtain $\xi_{\rm obs}(r, 0, 2)/\xi(r, 0, 0) \simeq 0.224$ and $\xi_{\rm obs}(r, 0, 1)/\xi(r, 0, 0) \simeq 0.375$. In this regime, we find that the approximation for $\xi_{\rm obs}$ introduced by MCLM is accurate to $2 - 3\%$.

In order to extend our analysis also to the mildly non–linear evolution, we numerically computed $\xi_{\rm obs}$ by using the cross correlation given in equation (11). The result obtained for $[z_{\rm min}, z_{\rm max}] = [0, 2]$ is shown in Fig. 8: also in this case $\xi_{\rm obs}$ looks like the usual correlation function evaluated at some intermediate redshift. We then tested the accuracy of the above mentioned simplified scheme for the computation of $\xi_{\rm obs}$, finding good agreement between the exact and the rough estimates (excluding a small neighbourhood of the zero–crossing radius of $\xi(r, z_1, z_2)$, where the approximated method breaks down, we find a maximum discrepancy of 6% for $[z_{\rm min}, z_{\rm max}] = [0, 2]$ and of 3% for $[z_{\rm min}, z_{\rm max}] = [0, 1]$). Anyway, the simplified procedure to compute $\xi_{\rm obs}$ can be further improved: adopting a different way of performing the average between redshifts, namely $1 + \bar{z} = [(1 + z_1)(1 + z_2)]^{1/2}$, ensures more accurate predictions (in this case the maximum error is always of the order of 1%). Probably this higher precision is due to the fact that we are considering mildly non–linear scales and the latter approximation gives exact results for linear evolution.

6 SUMMARY

In this paper, we have studied in detail the evolution of the mass two–point correlation function by describing the growth of density perturbations through ZA. Our motivations were originated by the well known ability of ZA to reproduce the weakly non–linear regime of gravitational dynamics. On scales that are not affected by shell–crossing, we found that the correlation function steepens as the clustering amplitude increases. Moreover, we showed that non–linear interactions are able to move the first zero crossing of $\xi(r)$ and we gave a quantitative description of this shifting for a CDM linear spectrum.

We then compared our results with the predictions of the scaling ansatz for clustering evolution formulated by JMW, obtaining remarkable agreement between the correlations on mildly non–linear scales and on completely linear scales. However, between these two regimes, the JMW prescription, which has been obtained requiring the resulting correlation to reproduce the linear behaviour on large scales, predicts smaller clustering amplitudes than ZA. We think that this disagreement is caused by the smallness of the box used by JMW to perform their N–body simulations. Actually, imposing to match the linear solution where $\bar{\xi}_{\rm L} \to 0$, without having any constraint from numerical data on quasi–linear scales, could alter the accuracy of the fitting function that embodies the scaling ansatz. In connection with this hypothesis, we compared ZA predictions on correlations with the output of a different scaling ansatz calibrated against large box simulations by BG. In effect, on large scales, the BG formula agrees better with ZA, keeping the same accuracy of the JMW fit on intermediate scales.

On the other hand, the reliability of ZA on these scales and for dynamically evolved fields ($\sigma_8 \gtrsim 1$) should be verified by directly comparing its predictions with the results of other approximations and numerical simulations.

Finally, we studied the evolution of the cross correlation between the density field evaluated at two different epochs and, adopting the method introduced by MCLM, we used our results to compute the theoretical prediction for the observed correlation function deriving from a deep catalogue of objects. In this context, we proposed a simplified procedure for the computation of $\xi_{\rm obs}$ that, at least for quasi–linear scales, significantly improves another approximation previously introduced by MCLM. This result confirms that the MCLM method can be used to make quantitative predictions about clustering evolution that find a direct observative counterpart in the analysis of deep surveys.

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